2.7 Prove Angle Pair Relationships

Before You identified relationships between pairs of angles.
Now You will use properties of special pairs of angles.
Why? So you can describe angles found in a home, as in Ex. 44.

Key Vocabulary
- **complementary angles,** *p. 35*
- **supplementary angles,** *p. 35*
- **linear pair,** *p. 37*
- **vertical angles,** *p. 37*

Sometimes, a new theorem describes a relationship that is useful in writing proofs. For example, using the *Right Angles Congruence Theorem* will reduce the number of steps you need to include in a proof involving right angles.

THEOREM *For Your Notebook*

THEOREM 2.3 Right Angles Congruence Theorem

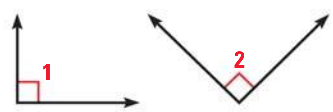
All right angles are congruent.

Proof: below

PROOF Right Angles Congruence Theorem

WRITE PROOFS
When you prove a theorem, write the hypothesis of the theorem as the GIVEN statement. The conclusion is what you must PROVE.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles.
PROVE ▶ $\angle 1 \cong \angle 2$



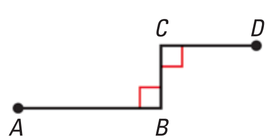
STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

EXAMPLE 1 Use right angle congruence

AVOID ERRORS
The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

Write a proof.

GIVEN ▶ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$
PROVE ▶ $\angle B \cong \angle C$

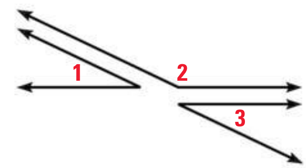


STATEMENTS	REASONS
1. $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$	**1.** Given
2. $\angle B$ and $\angle C$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle B \cong \angle C$	**3.** Right Angles Congruence Theorem

THEOREMS — *For Your Notebook*

THEOREM 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

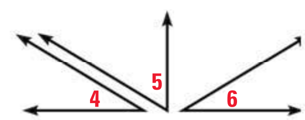


If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Proof: Example 2, below; Ex. 36, p. 129

THEOREM 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.



If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof: Ex. 37, p. 129; Ex. 41, p. 130

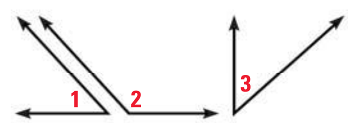
To prove Theorem 2.4, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of Theorem 2.5 also requires two cases.

EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 2$ are supplements.

PROVE ▶ $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 2$ are supplements.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 2 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** Definition of congruent angles

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✓ GUIDED PRACTICE for Examples 1 and 2

1. How many steps do you save in the proof in Example 1 by using the *Right Angles Congruence Theorem*?

2. Draw a diagram and write GIVEN and PROVE statements for a proof of each case of the *Congruent Complements Theorem*.

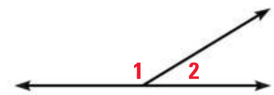
INTERSECTING LINES When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you used in Lesson 1.5 for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used in the proof of the *Vertical Angles Congruence Theorem*.

POSTULATE — For Your Notebook

POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

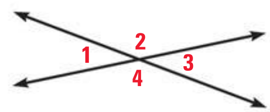
$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



THEOREM — For Your Notebook

THEOREM 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



Proof: Example 3, below

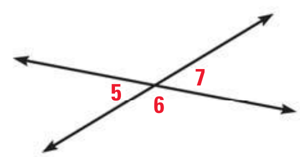
$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN ▶ $\angle 5$ and $\angle 7$ are vertical angles.

PROVE ▶ $\angle 5 \cong \angle 7$

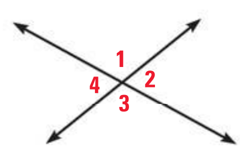


STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	**1.** Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	**2.** Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	**3.** Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	**4.** Congruent Supplements Theorem

USE A DIAGRAM
You can use information labeled in a diagram in your proof.

✓ GUIDED PRACTICE for Example 3

In Exercises 3–5, use the diagram.



3. If $m\angle 1 = 112°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

4. If $m\angle 2 = 67°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.

5. If $m\angle 4 = 71°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

6. Which previously proven theorem is used in Example 3 as a reason?

★ EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES
Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

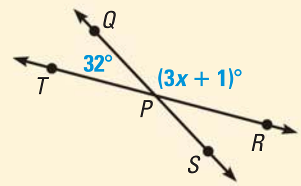
Which equation can be used to find x?

Ⓐ $32 + (3x + 1) = 90$

Ⓑ $32 + (3x + 1) = 180$

Ⓒ $32 = 3x + 1$

Ⓓ $3x + 1 = 212$



Solution

Because $\angle TPQ$ and $\angle QPR$ form a linear pair, the sum of their measures is 180°.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

Use the diagram in Example 4.

7. Solve for x.

8. Find $m\angle TPS$.

2.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 30, and 45

SKILL PRACTICE

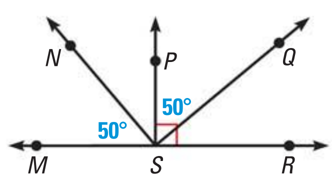
1. VOCABULARY Copy and complete: If two lines intersect at a point, then the _?_ angles formed by the intersecting lines are congruent.

2. ★ WRITING *Describe* the relationship between the angle measures of complementary angles, supplementary angles, vertical angles, and linear pairs.

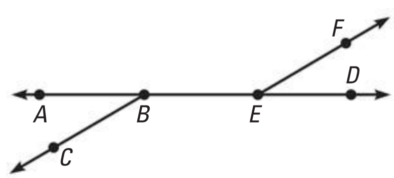
EXAMPLES 1 and 2 on pp. 124–125 for Exs. 3–7

IDENTIFY ANGLES **Identify the pair(s) of congruent angles in the figures below. *Explain* how you know they are congruent.**

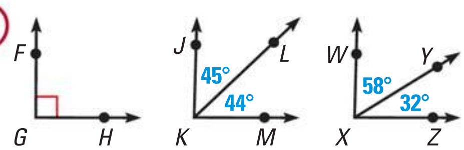
3.



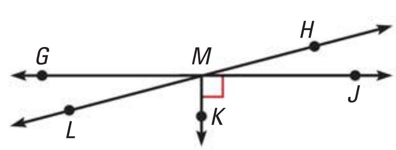
4. $\angle ABC$ is supplementary to $\angle CBD$. $\angle CBD$ is supplementary to $\angle DEF$.



5.



6.



7. ★ **SHORT RESPONSE** The x-axis and y-axis in a coordinate plane are perpendicular to each other. The axes form four angles. Are the four angles congruent right angles? *Explain.*

EXAMPLE 3
on p. 126
for Exs. 8–11

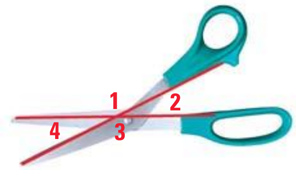
FINDING ANGLE MEASURES In Exercises 8–11, use the diagram at the right.

8. If $m\angle 1 = 145°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

9. If $m\angle 3 = 168°$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

10. If $m\angle 4 = 37°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

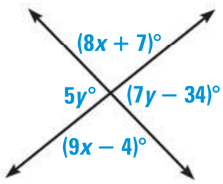
11. If $m\angle 2 = 62°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.



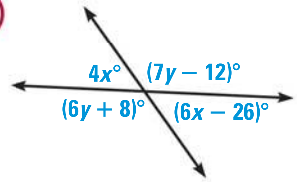
EXAMPLE 4
on p. 127
for Exs. 12–14

ALGEBRA Find the values of x and y.

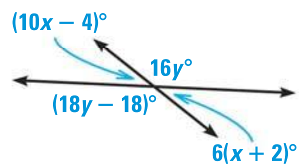
12.



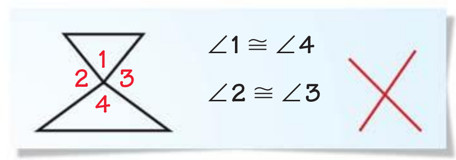
13.



14.



15. **ERROR ANALYSIS** *Describe* the error in stating that $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$.



16. ★ **MULTIPLE CHOICE** In a figure, $\angle A$ and $\angle D$ are complementary angles and $m\angle A = 4x°$. Which expression can be used to find $m\angle D$?

Ⓐ $(4x + 90)°$ Ⓑ $(180 - 4x)°$ Ⓒ $(180 + 4x)°$ Ⓓ $(90 - 4x)°$

FINDING ANGLE MEASURES In Exercises 17–21, copy and complete the statement given that $m\angle FHE = m\angle BHG = m\angle AHF = 90°$.

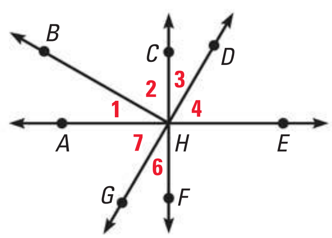
17. If $m\angle 3 = 30°$, then $m\angle 6 =$ __?__.

18. If $m\angle BHF = 115°$, then $m\angle 3 =$ __?__.

19. If $m\angle 6 = 27°$, then $m\angle 1 =$ __?__.

20. If $m\angle DHF = 133°$, then $m\angle CHG =$ __?__.

21. If $m\angle 3 = 32°$, then $m\angle 2 =$ __?__.



ANALYZING STATEMENTS Two lines that are not perpendicular intersect such that $\angle 1$ and $\angle 2$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 3$ are vertical angles. Tell whether the statement is true.

22. $\angle 1 \cong \angle 2$

23. $\angle 1 \cong \angle 3$

24. $\angle 1 \cong \angle 4$

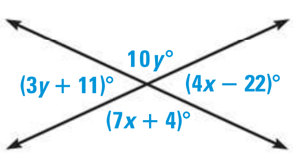
25. $\angle 3 \cong \angle 2$

26. $\angle 2 \cong \angle 4$

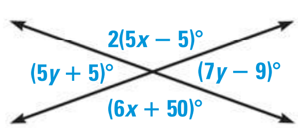
27. $m\angle 3 + m\angle 4 = 180°$

ALGEBRA Find the measure of each angle in the diagram.

28.



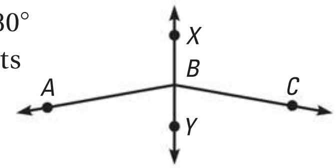
29.



◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

30. ★ **OPEN-ENDED MATH** In the diagram, $m\angle CBY = 80^\circ$ and $\overleftrightarrow{XY}$ bisects $\angle ABC$. Give two more true statements about the diagram.



DRAWING CONCLUSIONS **In Exercises 31–34, use the given statement to name two congruent angles. Then give a reason that justifies your conclusion.**

31. In triangle GFE, $\overrightarrow{GH}$ bisects $\angle EGF$.

32. $\angle 1$ is a supplement of $\angle 6$, and $\angle 9$ is a supplement of $\angle 6$.

33. $\overline{AB}$ is perpendicular to $\overline{CD}$, and $\overline{AB}$ and $\overline{CD}$ intersect at E.

34. $\angle 5$ is complementary to $\angle 12$, and $\angle 1$ is complementary to $\angle 12$.

35. **CHALLENGE** Sketch two intersecting lines j and k. Sketch another pair of lines ℓ and m that intersect at the same point as j and k and that bisect the angles formed by j and k. Line ℓ is perpendicular to line m. *Explain* why this is true.

PROBLEM SOLVING

EXAMPLE 2
on p. 125
for Ex. 36

36. **PROVING THEOREM 2.4** Prove the second case of the Congruent Supplements Theorem where two angles are supplementary to congruent angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.
$\angle 1 \cong \angle 4$

PROVE ▶ $\angle 2 \cong \angle 3$

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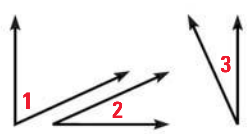
37. **PROVING THEOREM 2.5** Copy and complete the proof of the first case of the Congruent Complements Theorem where two angles are complementary to the same angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are complements.
$\angle 1$ and $\angle 3$ are complements.

PROVE ▶ $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements.	**1.** ?
2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$	**2.** ?
3. ?	**3.** Transitive Property of Equality
4. ?	**4.** Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	**5.** ?

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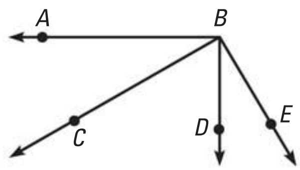
PROOF **Use the given information and the diagram to prove the statement.**

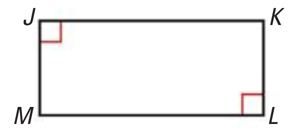
38. **GIVEN** ▶ $\angle ABD$ is a right angle.
$\angle CBE$ is a right angle.

PROVE ▶ $\angle ABC \cong \angle DBE$



39. **GIVEN** ▶ $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
$\angle J \cong \angle M$, $\angle K \cong \angle L$

PROVE ▶ $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



40. **MULTI-STEP PROBLEM** Use the photo of the folding table.

a. If $m\angle 1 = x°$, write expressions for the other three angle measures.

b. Estimate the value of x. What are the measures of the other angles?

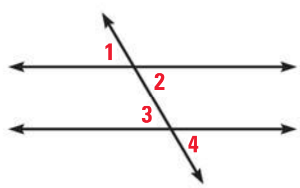
c. As the table is folded up, $\angle 4$ gets smaller. What happens to the other three angles? *Explain* your reasoning.

41. **PROVING THEOREM 2.5** Write a two-column proof for the second case of Theorem 2.5 where two angles are complementary to congruent angles.

WRITING PROOFS **Write a two-column proof.**

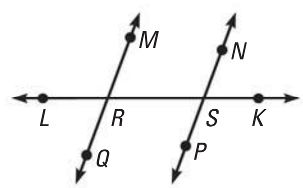
42. **GIVEN** ▶ $\angle 1 \cong \angle 3$

PROVE ▶ $\angle 2 \cong \angle 4$



43. **GIVEN** ▶ $\angle QRS$ and $\angle PSR$ are supplementary.

PROVE ▶ $\angle QRL \cong \angle PSR$



44. **STAIRCASE** Use the photo and the given information to prove the statement.

GIVEN ▶ $\angle 1$ is complementary to $\angle 3$.
$\angle 2$ is complementary to $\angle 4$.

PROVE ▶ $\angle 1 \cong \angle 4$

45. ★ **EXTENDED RESPONSE** $\angle STV$ is bisected by $\overrightarrow{TW}$, and $\overrightarrow{TX}$ and $\overrightarrow{TW}$ are opposite rays. You want to show $\angle STX \cong \angle VTX$.

a. Draw a diagram.

b. Identify the GIVEN and PROVE statements for the situation.

c. Write a two-column proof.

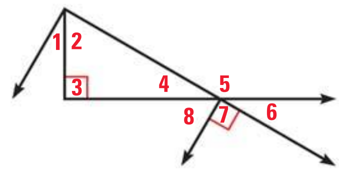
46. **USING DIAGRAMS** Copy and complete the statement with <, >, or =.

a. $m\angle 3$ _?_ $m\angle 7$

b. $m\angle 4$ _?_ $m\angle 6$

c. $m\angle 8 + m\angle 6$ _?_ $150°$

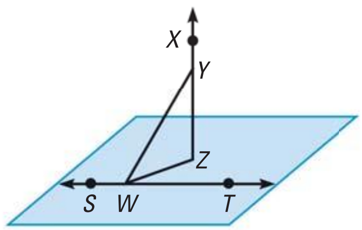
d. If $m\angle 4 = 30°$, then $m\angle 5$ _?_ $m\angle 4$



CHALLENGE **In Exercises 47 and 48, write a two-column proof.**

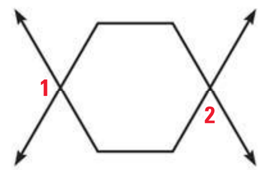
47. **GIVEN** ▶ $m\angle WYZ = m\angle TWZ = 45°$

PROVE ▶ $\angle SWZ \cong \angle XYW$



48. **GIVEN** ▶ The hexagon is regular.

PROVE ▶ $m\angle 1 + m\angle 2 = 180°$



MIXED REVIEW

PREVIEW
Prepare for Lesson 3.1 in Exs. 49–52.

In Exercises 49–52, sketch a plane. Then sketch the described situation. *(p. 2)*

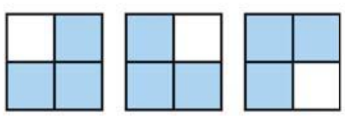
49. Three noncollinear points that lie in the plane

50. A line that intersects the plane at one point

51. Two perpendicular lines that lie in the plane

52. A plane perpendicular to the given plane

53. Sketch the next figure in the pattern. *(p. 72)*



QUIZ *for Lessons 2.6–2.7*

Match the statement with the property that it illustrates. *(p. 112)*

1. If $\overline{HJ} \cong \overline{LM}$, then $\overline{LM} \cong \overline{HJ}$.

2. If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 4$, then $\angle 1 \cong \angle 4$.

3. $\angle XYZ \cong \angle XYZ$

A. Reflexive Property of Congruence

B. Symmetric Property of Congruence

C. Transitive Property of Congruence

4. Write a two-column proof. *(p. 124)*

GIVEN ▶ $\angle XWY$ is a straight angle.
$\angle ZWV$ is a straight angle.

PROVE ▶ $\angle XWV \cong \angle ZWY$